GRUPPI CHE HANNO LO STESSO OLOMORFO

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Given a group G, the normaliser $N_{S(G)}(\rho(G))$ of the image $\rho(G)$ of its right regular representation in the group $S(G)$ of permutations on the set G is isomorphic to the holomorph of G, that is, to the natural split extension of G by its automorphism group. More generally, if N is a regular subgroup of $S(G)$, its normaliser $N_{S(G)}(N)$ will be isomorphic to the holomorph of N. It is then natural to say that G and N have the same holomorph when the respective normalisers coincide, that is,

$$N_{S(G)}(\rho(G)) = N_{S(G)}(N).$$

We will discuss

- the case [CDV17] when G is abelian and finitely generated, where we have redone work of Mills [Mil51] via associated rings, and
- the case [CDV18] when G is finite and perfect, which leads among others to the following question, which appears to be still open:

  > Is there a finite, quasisimple group Q such that the centre $Z(Q)$ of Q is not an elementary abelian 2-group, and the automorphism group of Q acts trivially on $Z(Q)$?

(Both cases are joint work with Francesca Dalla Volta.)

We will then discuss the case [Car18] when G is a finite p-group of nilpotence class two. Here we consider the group

$$T(G) = N_{S(G)}(N_{S(G)}(\rho(G)))/N_{S(G)}(\rho(G))),$$

(the group $N_{S(G)}(N_{S(G)}(\rho(G)))$ is referred to as the multiple holomorph of G in the literature), that acts regularly on the set of the regular subgroups N which have the same holomorph as G, and are isomorphic to G, and show that its structure can be somewhat intricate.

The problems we consider can be reformulated in terms of skew braces.

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References

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