Dipartimento di Matematica e Informatica "U. Dini", Firenze

Seminario di Algebra - 2017/2018

1. 25 ottobre 2017. Eloisa Detomi [Padova], *Parole e ricoprimenti in gruppi profiniti.*

   Sia w una parola e G un gruppo profinito. Se l'insieme G_w di tutti i valori che w assume in G è contenuto nell'unione di un insieme finito (o numerabile) di sottogruppi G_i di G, le proprietà dei sottogruppi G_i possono influenzare la struttura del sottogruppo generato da G_w il sottogruppo verbale $w(G)$. Presenterò alcuni risultati ottenuti in collaborazione con M. Morigi e P. Shumyatsky, nel caso di ricoprimenti con sottogruppi pronilpotenti o localmente nilpotenti.

2. 5 novembre 2017. Pablo Spiga [MIlano - Bicocca], *On the classification of the finite primitive groups of small rank.*

   A permutation group is primitive if it leaves invariant no non-trivial partition. In a broad sense, primitive groups are building blocks for arbitrary permutation groups. The rank of a primitive group is the number of orbits of the stabiliser of a point. One of the earlier applications of the classification of the finite simple groups is the classification by Peter Cameron of the primitive groups of rank 2, that is, 2-transitive groups. Later, for some applications in finite geometry and on generalized polygons permutation group theorists have classified the primitive groups of rank 3. Now, for investigating some recent conjectures of Muzychuck on the permutation character of a primitive group, there is some interest in classifying the primitive groups of "small" rank, where small is not necessarily a constant. In the talk, we discuss some progress towards this classification.

3. 22 novembre 2017. Orazio Puglisi [Firenze], *Automorphisms of the rational Urysohn's space.*

   Rational Urysohn's space U is a "highly homogeneus" metric space with some pretty strong embedding properties. E.g. every countable rational metric space (i.e. a space on which the metric takes rational values) can be isometrically embedded in this space. In this talk I will try to describe some results obtained by Cameron and Vershik on the subgroups of $Aut(U)$, as well as some new results contained in the thesis of Clarissa Fanfani.

4. 30 novembre 2017. Chandan Dalawat [Harish-Chandra Research Institute, Allahabad (India)], *Solvable primitive extensions.*

   A finite separable extension E of a field F is called *primitive* if there are no intermediate extensions. It is called *solvable* if the group $Gal(\hat{E}|F)$ of automorphisms of its galoisian closure $\hat{E}$ over F is solvable. What are the solvable primitive extensions of F? This problem goes back to Évariste Galois; we show how some recent work complements his insights. We show that a solvable primitive extension E of F is uniquely determined (up to F-isomorphism) by $\hat{E}$ and characterise the extensions D of F such that $D = \hat{E}$ for some solvable primitive extension E of F. Not much mathematical background will be assumed.

5. 13 dicembre 2017. Silvio Dolfi [Firenze], *Some properties of the degree graph.*

   I will discuss a few results concerning the prime graph of the irreducible character degrees (degree graph, for short) of a finite group. I will start from a theorem by P. Palfy (stating that the independence number for the degree graph of a solvable group is at most 2) and then discuss some recent developments.

6. 20 dicembre 2017. Dmitry Malinin [Firenze], *On the arithmetic of integral representations of finite groups.*

   We discuss arithmetic aspects of integral representations of finite groups and some related questions: Schur groups, globally irreducible representations, quadratic lattices, Galois algebras, realization fields.

7. 24 gennaio 2018. Andrea Lucchini [Padova], *Maximal subgroups of a finite group that avoid a minimal set of generators.*

   We give an elementary proof of the following remark: if G is a finite group and $\{g_1, \dots, g_d\}$ is a generating set of G of smallest cardinality, then there exists a maximal subgroup M of G such that $M \cap \{g_1, \dots, g_d\} = \emptyset$. This result leads us to investigate the freedom that one has in the choice of the maximal subgroup M of G. We obtain information in this direction in the case when G is soluble, describing for example the structure of G when there is a unique choice for M. When G is a primitive permutation group one can ask whether is it possible to choose in the role of M a point-stabilizer. We give a positive answer when G is a 3-generated primitive permutation group but we leave open the following question: does there exist a (soluble) primitive permutation group $G = \langle g_1, \dots, g_d \rangle$ with $d(G) = d > 3$ and with $\bigcap_{1 \leq i \leq d} supp(g_i) = \emptyset$? We obtain a weaker result in this direction: if $G = \langle g_1, \dots, g_d \rangle$ with $d(G) = d$, then $supp(g_i) \cap supp(g_j) \neq \emptyset$ for all $i, j \in \{1, \dots, d\}$.

8. 31 gennaio 2018. Marta Morigi [Bologna], *Conciseness of words of Engel type in residually finite groups.*

   Given a group-word w and a group G, the verbal subgroup $w(G)$ is the one generated by all w-values in G. The word w is said to be concise if $w(G)$ is finite whenever the set of w-values in G is finite. In the sixties P. Hall asked whether every word is concise but later Ivanov answered this question in the negative. On the other hand, Hall's question remains wide open in the class of residually finite groups. In this talk we show that various generalizations of the Engel word are concise in residually finite groups.

9. 6 febbraio 2018 Francesco Matucci [Campinas - Brasile], *Introduction to Geometric Group Theory I.*

10. 7 febbraio 2018. Francesco Matucci [Campinas - Brasile], *Introduction to Geometric Group Theory II.*

    Geometric Group Theory sees groups as symmetries of a space on which they act on. The goal is to find connections between the algebraic structure of groups, the geometry and topology of the associated space and the dynamics of the action. Starting from an abstract group, we can always associate a geometric object (known as the Cayley graph) which realizes this natural symmetry. The goal of these talks is to introduce some concepts used to analyze groups (such as that of quasi-isometry) and some interesting classes of groups whose definition or proof methods border in other areas of Math (such as geometry, combinatorics, computer science, analysis, topology and more).

11. 14 febbraio 2018. Francesco Matucci [Campinas - Brasile],*Rational Embeddings of Hyperbolic Groups.*

    For a finitely generated group, the Cayley graph is a metric space encoding the structure of the group. Gromov introduced the notion of a δ-hyperbolic group, a finitely generated group with a negatively curved Cayley graph, that is, for any triangle in the graph with geodesic sides, each side is contained in the δ-neighborhood of the union of the two other sides. Hyperbolic groups are "prevalent" among finitely generated groups. Grigorchuk, Nekrashevych and Sushchanskii defined the rational group as the full group of homeomorphisms of a Cantor space and which admit precisely finitely many types of "local actions" described by finite state transducers (one of many models of computing machines). This is a rather large group and, by construction, it contains all groups generated by finite state automata (for example, the Grigorchuk group of intermediate word growth). In this talk I will introduce these groups and some of their properties and explain how to embed a class of hyperbolic groups in the rational group. Parts of this talk are joint with James Belk (Bard College), Collin Bleak (University of St. Andrews) and James Hyde (University of St. Andrews).

12. 21 febbraio 2018. Francesco Fumagalli [Firenze], *Primes and Groups.*

    We will discuss upon some arithmetical functions related to the indices of subgroups in finite groups. Some interesting number theory problems, mostly dealing with the distribution of primes, arise quite naturally in this study. Part of the talk will concern maximal subgroups in finite simple groups.

13. 28 febbraio 2018. Antongiulio Fornasiero [Firenze], *Matroids and entropy.*

    I will give an introduction to matroids, and then describe the entropy of their endomorphisms.

14. 7 marzo 2018. Ilaria Del Corso [Pisa], *On Fuchs' problem about the group of units of a ring.*

    L. Fuchs in [Abelian groups, 3rd edn (Pergamon, Oxford, 1960); Problem 72] posed the following problem: *Characterize the groups which are the groups of all units in a commutative and associative ring with identity.*
    A partial approach to this problem was suggested by Ditor in 1971, with the following less general question: *Which whole numbers can be the number of units of a ring?* In the following years, these questions inspired the work of many authors, and some partial answer to them has been given. Among the others, we recall the work by Gilmer (1963), Hallett, Hirsch and Zassenhauss (1965-66), Pearson and Schneider (1970), Dolzan (2002) and the recent papers by Chebolu and Lockridge (2015-17).
    Recently, in two joint papers with R. Dvornicich, we studied the original Fuchs question and we almost answered it. In fact, we have been able to obtain a pretty good description of the possible groups of units equipped with families of examples of both realizable and non-realizable groups. We also examined the interesting case of torsion-free rings and we completely classified the possible finite abelian groups of units which arrise in this case. As a consequence of our results we completely answered Ditor's question on the possible cardinalities of the group of units of a ring.

15. 14 marzo 2018. Collin Bleak [St. Andrews - U.K.], *Generalised Ping-Pong Lemmas, and the group of PL homeomorphisms of the unit interval.*

    We define a notion of fast generating sets, for groups of self-homeomorphisms of a space (under composition), and apply it to the group $Pl_o(I)$ of piecewise-linear homeomorphisms of the unit interval. As a consequence, we build some general forms of Ping-Pong Lemmas for this group, which lemmas guarantee isomorphism types for certain finitely generated subgroups of $Pl_o(I)$, based on simple combinatorial data. We also find a lemma which guarantees that some particular (unexpectedly large) set of subgroups of $Pl_o(I)$ also embed in R. Thompsons group F. Joint with Matt Brin and Justin Moore.

16. 21 marzo 2018. Zeinab Akhlaghi [Tehran], *A dual version of Huppert's conjecture on conjugacy class sizes.*

    In 2000, Huppert posed the following conjecture: Let G be a group and let H be a nonabelian simple group. If $cd(G) = cd(H)$, then $G = H \times A$, where A is an abelian group. In this talk we are going to investigate a dual of Huppert's conjecture on conjugacy class sizes of some simple groups.
    With another point of view, the dual of Huppert's conjecture is an extension of Thompson's conjecture, which says: Every finite group G with the property $Z(G) = 1$ and $cs(G) = cs(S)$ is isomorphic to S where $cs(G)$ is the set of conjugacy class sizes of G and S is a simple group. Thompsons conjecture is proved for a lot of simple groups using CFSG. We prove the dual of Huppert's conjecture for $PSL_2(q)$ and $Suz(q)$ without using CFSG.

17. 4 aprile 2018. Lucia Sanus [Valencia], *Sylow Subgroups and Fusion of Characters.*

    Suppose that P is a Sylow p-subgroup of the finite group G. We are interested in the set $\mathrm{Cl}_G(P)$ consisting of the conjugacy classes x^P of P such that $x^G \cap P = x^P$, and in the set $\mathrm{Irr}_G(P)$ consisting of the irreducible characters α of P such that $\alpha(x) = \alpha(y)$ whenever $x, y \in P$ are G-conjugate. Of course, when $P \trianglelefteq G$, we have that $\mathrm{Cl}_G(P)$ is the set of G-invariant classes of P, and $\mathrm{Irr}_G(P)$ is the set of G-invariant irreducible characters of P.

18. 11 aprile 2018. Emanuele Pacifici [Milano], *Supercharacters of finite groups.*

    The concept of "supercharacter theory" for a finite group has been introduced by I.M. Isaacs and P. Diaconis in 2008 ([2]), as a generalization of an idea that proved effective in the study of groups of upper unitriangular matrices over a finite field: while, in fact, the explicit determination of the irreducible characters and conjugacy classes is in general a difficult problem for these groups, it is possible to develop for them an elegant supercharacter theory ([1]). Supercharacter theories subsequently found applications in several contexts, among which number theory, random walks, Fourier analysis on finite groups.
    Recent papers analyze the possible supercharacter theories in particular classes of groups (e.g. cyclic groups, extraspecial p-groups and Frobenius groups); others aim at classifying all finite groups whose supercharacter theories satisfy certain given conditions. In the talk, we will discuss some aspects of this research area.

    [1] C. André, Basic characters of the unitriangular group, J. Algebra 175 (1995), 287-319.
    [2] I.M. Isaacs, P. Diaconis, Supercharacters and superclasses for algebra groups, Trans. Amer. Math. Soc. 360 (2008), 2359-2392.

19. 18 aprile 2018. Víctor Manuel Ortiz-Sotomayor [Valencia], *Impact of conjugacy class sizes on factorised groups.*

    The relationship between some subsets of the class sizes of a finite group and its structure has been quite analysed by many authors. In parallel with this research, the study of groups factorised as the product of two subgroups has been gaining an increasing interest. The aim of this talk is to report about new progress on the structure of a factorised group when arithmetical properties are imposed on the class sizes of certain elements in the factors..

20. 16 maggio 2018. Noa Lavi [Open University - Israele], *Positivity and boundedness in ordered valued fields.*

    We give a criterion for rational functions to be positive\bounded on a generalized semi-algebraic set: that is, a set defined by inequalities and valuative inequalities, giving a generalization and analogue to the Hilbert 17th problem. We use tools from model theory of valued fields.

21. 23 maggio 2018. Pavel Shumyatsky [Brasilia], *Profinite groups with abelian centralizers.*

    I will present a theorem that if G is a profinite group in which all centralizers are abelian, then G is either virtually abelian or virtually pro-p. This is a joint result with Pavel Zalesski and Theo Zapata.

22. 30 maggio 2018. Anupam Singh [I.I.S.E.R. - Pune, India], *Word-maps on groups.*

    Solving equations is a fundamental problem in Mathematics. We look at the equation $w(x_1, ..., x_n) = g$ where w is a monomial in the variables $x_1, ..., x_n$ for an element g in a group G. In this talk I will begin with a brief survey of progress made to solve this problem over finite simple groups. I will also report on my joint work with Amit Kulshrestha where we study power maps for $SL(2, k)$ and reduce the problem to solving certain polynomial equations involving generalized Fibonacci polynomials.

23. 6 giugno 2018. Luca Sabatini [Firenze], *Bipartite Ramanujan graphs.*

    A d-regular graph is called a Ramanujan graph if all non-trivial eigenvalues of the adjacency matrix of Γ lie between $-2\sqrt{d-1}$ and $2\sqrt{d-1}$. Ramanujan graphs are thus the best 'expanders' (regular graphs that are both sparse and highly connected). For instance, complete graphs are Ramanujan, but the hard problem is to find arbitrary large Ramanujan graphs for a fixed valence d. In the seminar, I will report on the recent construction [1], by Marcus, Spielman and Srivastava, of infinite families of d-regular bipartite Ramanujan graphs, for every $d > 2$. The proof uses the concept of a 2-lift of a graph and exploits a recent technique for controlling the eigenvalues of certain random matrices, called the method of "interlacing polynomials".

    [1] A.W. Marcus, D.A. Spielman, N. Srivastava. *Interlacing families I: bipartite Ramanujan graphs of all degrees.* Ann. of Math. 182 (2015), 307–325.

24. 13 giugno 2018. Zeinab Araghi Rostami [Mashhad - Iran], *The Bogomolov multiplier of Lie algebras.*

    The Bogomolov multiplier is a group-theoretical invariant introduced in the study of the rationality problem in algebraic geometry. Let $\mathbb{K}$ be a field, G a finite group and V a faithful representation of G over K. Then there is natural action of G upon the field of rational functions $\mathbb{K}(V)$. The rationality problem (also known as Noether's problem) asks whether the field of G-invariant functions $\mathbb{K}(V)^G$ is purely transcendental over $\mathbb{K}$. Define

$$B_0(G) = \bigcap\{\ker\{res_G^A : H^2(G, \mathbb{Q}/\mathbb{Z}) \longrightarrow H^2(A, \mathbb{Q}/\mathbb{Z})\}$$

where A runs among all the abelian subgroups of G. The group $B_0(G)$ is a subgroup of the Schur multiplier $M(G) = H^2(G, \mathbb{Q}/\mathbb{Z})$ and Kunyavskii named it the *Bogomolov multiplier* of G. Vanishing of the Bogomolov multiplier leads to positive answer to Noether's problem. But it is not always easy to calculate Bogomolov multipliers of groups. Recently, after introducing the concept of curly exterior product, Moravec provides a new equivalent definition for the Bogomolov multiplier.

We define Bogomolov multipliers for Lie algebras and show that, if G is a finite p-group of nilpotency class less than p, and L is the Lie ring corresponding to G via the Lazard correspondence, the Bogomolov multipliers of G and L are isomorphic as abelian groups.